

Radiative vertex $VP\gamma$ and $\eta - \eta'$ mixing in light-cone sum rules

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In this work, we calculate radiative vertex $VP\gamma$ ($V = \phi, \omega, \rho$ and $P = \eta, \eta'$) by utilizing $\omega - \phi$ mixing scheme and taking into account the contributions of the three-particle twist-4 distribution amplitudes of the photon in QCD sum rules on light cone. According to experimental data of $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ from PDG, a value of the $\eta - \eta'$ mixing angle, $\varphi = (40.9 \pm 0.5)^\circ$, is extracted in the framework of the quark-flavor basis to describe the $\eta - \eta'$ system.

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I. INTRODUCTION

The problem of $\eta - \eta'$ mixing in the pseudoscalar-meson nonet has been studied many times in the last forty years. Because those researches play a important role for the $SU(3)$ -breaking effect and the $U(1)_A$ anomaly. In contrast to the ϕ and ω mesons, where they are taken as almost ideally mixed states with quark content of well defined flavor, $\eta - \eta'$ mixing is still a debated subject. In the pioneering work[1–3], a mixing angle θ_P was conventionally introduced to describe η and η' as linear combinations of octet and singlet basis states. With the development of the experiment, a phenomenological investigation [4] found that one mixing angel was insufficient to describe more physical processes, where η or η' meson appears in the initial state or the final state. And then two new equivalent schemes to describe the $\eta - \eta'$ mixing was proposed by Leutwyler[5] and Feldmann et al.[6], respectively. The correspondence mixing angles are θ_1, θ_8 for the octet-singlet basis and φ_q, φ_s for the quark-flavor basis. More literatures about two mixing angles can be found in Refs.[8–12], where θ_1 is not equal θ_8 , but φ_q is equal φ_s apart from terms which violate the Okubo-Zweig-Iizuka (OZI) rule[7]. In our next calculation, we chose $\varphi_q = \varphi_s = \varphi$ in the quark-flavor scheme to investigate radiative vertex $VP\gamma$ ($V = \phi, \omega, \rho$ and $P = \eta, \eta'$).

The radiative decays between light pseudoscalar (P) and light vector (V) mesons are an excellent laboratory for investigating the nature and extracting the non-perturbative parameters of light pseudoscalar nonet in low-energy hadron physics. Among the characteristics of the electromagnetic interaction processes, the coupling constant, $g_{VP\gamma}$, plays one of the most important roles, since they determine the strength of the hadron interactions. So by investigating the above six radiative vertexes, in which η or η' meson was involved, one can extracting the mixing angle of $\eta - \eta'$ system. Interest on this issue has been performed by many authors, for example, in Refs.[6, 9, 13–16]. In this paper, we renewed this issue in QCD sum rules on light cone. Since the radiative decays of light meson is belong to the low-energy hadron interaction, which is governed by non-perturbative QCD, it is very difficult to obtain the numerical values of the coupling constants from the first principles. In order to interpret coupling constants from the experimental data, we immediately need to deal with large distance effects from the photon besides the hadrons, because a special feature of the QCD description of hard exclusive processes involving photon emission is that a real photon contains both a hard electromagnetic and a soft hadronic component. In Ref.[17, 18], a consistent technique was proposed by closed analogy with distribution amplitudes (DAs) of mesons [19, 20]. The soft hadronic components of the photon are related to matrix elements of light-cone operators with different twist in the electromagnetic background field and can be parameterized in terms of photon DAs. Since the photon emission from the light quark takes place at large distances, the use of standard QCD sum rules based on the local operator product expansion (OPE) is not sufficient. Rather, one should use a light-cone expansion which is adequate for exclusive processes with light particles.

The paper is organized as follows. In section 2, we derive the coupling constants $g_{VP\gamma}$ by utilizing $\omega - \phi$ mixing scheme and taking into account the contributions of the three-particle twist-4 distribution amplitudes of the photon in the light-cone sum rules. In section 3, we present our numerical analysis. The final section is reserved for a conclusion. The photon distribution amplitudes are list in Appendix A and the overlap amplitudes for pseudoscalar mesons and vector mesons, which were defined in section 2, are presented in Appendix B.

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II. RADIATIVE VERTEX $VP\gamma$ IN LIGHT-CONE SUM RULES

In the framework of light cone QCD sum rules, we immediately choose the two point correlation function with the photon as follow

$$T_\mu(p, q) = i \int d^4x e^{-iqx} \langle \gamma(p) | T \{ j_\mu^V(x) j_5^P(0) \} | 0 \rangle, \quad (1)$$

to extract the radiative coupling constant $g_{VP\gamma}$. Here $j_\mu^V(x)$ is the vector meson current and $j_5^P(0)$ is the pseudoscalar current. According to the quark model of hadron and neglecting the contribution from the high Fock state of hadron, the above interpolating currents may be written as

$$j_\mu^\phi = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) \sin \beta + \bar{s}\gamma_\mu s \cos \beta, \quad j_\mu^\omega = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) \cos \beta - \bar{s}\gamma_\mu s \sin \beta, \quad j_\mu^\rho = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) \quad (2)$$

for light vector mesons ϕ, ω, ρ and

$$j_5^\eta = \frac{1}{\sqrt{2}}(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d) \cos \varphi - \bar{s}i\gamma_5 s \sin \varphi, \quad j_5^{\eta'} = \frac{1}{\sqrt{2}}(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d) \sin \varphi + \bar{s}i\gamma_5 s \cos \varphi \quad (3)$$

for light pseudoscalar mesons η, η' , respectively. Here φ is the value of the $\eta - \eta'$ mixing angle, which will be discussed in this work, and β is the value of the $\omega - \phi$ mixing angle, which has been determined from the available experimental data in the Ref.[21] as $\beta = 3.18^\circ$.

According to the basic assumption of quark-hadron duality in the QCD sum rules approach, we can insert two complete series of intermediate states with the same quantum numbers into the correlation function T_μ to obtain the hadronic representation. After isolating the contribution of the ground state by the pole terms of the vector meson and the pseudoscalar meson, we get the following result

$$T_\mu(p, q) = \frac{\langle P(-q)\gamma(p) | V(p-q) \rangle \langle V(p-q) | j_\mu^V | 0 \rangle \langle 0 | j_5^P | P(-q) \rangle}{((p-q)^2 - m_V^2)((-q)^2 - m_P^2)} + \dots, \quad (4)$$

for $V \rightarrow P\gamma$ decay, where $-q, p$ and $p-q$ denote the pseudoscalar meson, the photon and the vector meson momentum, respectively. The amplitudes of these interpolating currents with the meson states are defined as

$$\langle 0 | j_\mu^V | V(p-q) \rangle = \lambda_V u_\mu^V \quad (5)$$

$$\langle 0 | j_5^P | P(-q) \rangle = \lambda_P, \quad (6)$$

where u_μ^V is the polarization vector of the vector meson, λ_V and λ_P are called the overlap amplitudes which can be determined by QCD sum rules method in Appendix B. The coupling constant $g_{VP\gamma}$ is defined through the effective Lagrangian

$$\mathcal{L} = -\frac{e}{m_V} g_{VP\gamma} \varepsilon_{\mu\nu\alpha\beta} (\partial^\mu \phi_V^\nu - \partial^\nu \phi_V^\mu) (\partial^\alpha A^\beta - \partial^\beta A^\alpha) \phi_P \quad (7)$$

where ϕ_V, ϕ_P and A denote the vector field, the pseudoscalar field and the photon field, respectively. Therefore, the $\langle P(-q)\gamma(p) | V(p-q) \rangle$ matrix in the hadronic representation can be written as

$$\langle P(-q)\gamma(p) | V(p-q) \rangle = -\frac{e}{m_V} g_{VP\gamma} K(p^2) \varepsilon^{\mu\nu\alpha\beta} u_\mu^V q^\nu \varepsilon_\alpha^\gamma p^\beta, \quad (8)$$

where $p^2 = 0$ for the momentum of the real photon and $K(p^2)$ is a form factor with $K(0) = 1$. Substituting eq.(5),(6) and (8) into the hadronic representation, we obtain the physical part and choose the structure $\varepsilon_\mu^{\nu\alpha\beta} p_\nu q_\alpha \varepsilon_\beta^\gamma$ from which the corresponding invariant amplitude,

$$T((p-q)^2, q^2) = \frac{e g_{VP\gamma} \lambda_P \lambda_V}{m_V ((p-q)^2 - m_V^2) (q^2 - m_P^2)} + \int_\Sigma \frac{\rho^h(s_1, s_2) ds_1 ds_2}{(s_1 - q^2) (s_2 - (p-q)^2)} + \text{subtractions}. \quad (9)$$

The first term is the contribution of the ground-state and contains the $g_{VP\gamma}$ coupling, while the hadronic spectral function $\rho^h(s_1, s_2)$ represents the contribution of higher resonances and continuum states. The integration region in the (s_1, s_2) - plane is denoted by Σ and one may take $(s_1)^a + (s_2)^a \leq (s_0)^a$, where s_0 is the effective threshold in the

double dispersion relation. It is relevant with s_0^V and s_0^P , which are the effective thresholds in the vector meson and the pseudoscalar meson channels, respectively. How to take their values will be discussed in the next section. The subtraction terms isn't considered in the sum rules, because they will be removed by a double Borel transformation.

Next, we calculate the correlation function from QCD side by using light cone operator product expansion method, in which we work with large momenta, i.e., $-q^2$ and $-(p-q)^2$ are both large. The correlation function, then, can be calculated as an expansion near to the light cone $x^2 \approx 0$. The expansion involves matrix elements of the nonlocal operators between vacuum and the photon states in terms of the photon DAs with increasing twist. At the same time, the full quark propagator of the light quark [22, 23] in the presents of gluonic and electromagnetic background fields is used in this calculation, and it is given as

$$\begin{aligned} iS(x, 0) = & \langle 0 | T \{ \bar{q}(x) q(0) \} | 0 \rangle = i \frac{\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12} (1 + i \frac{m_q \not{x}}{4}) - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle (1 + i \frac{m_q \not{x}}{6}) \\ & - i g_s \int_0^1 dv \{ \frac{\not{x}}{16\pi^2 x^2} (\bar{\nu} \not{x} \sigma_{\mu\nu} + \nu \sigma_{\mu\nu} \not{x}) G^{\mu\nu}(vx) - i \frac{m_q}{32\pi^2} G^{\mu\nu}(vx) \sigma_{\mu\nu} \ln(\frac{-x^2 \Lambda^2}{4} + 2\gamma_E) \} \\ & - i e_q \int_0^1 dv \{ \frac{\not{x}}{16\pi^2 x^2} (\bar{\nu} \not{x} \sigma_{\mu\nu} + \nu \sigma_{\mu\nu} \not{x}) F^{\mu\nu}(vx) - i \frac{m_q}{32\pi^2} F^{\mu\nu}(vx) \sigma_{\mu\nu} \ln(\frac{-x^2 \Lambda^2}{4} + 2\gamma_E) \} + \dots, \quad (10) \end{aligned}$$

where $G_{\mu\nu}$ is the gluon field strength tensor and $F_{\mu\nu}$ is the electromagnetic field strength tensor. Taking into account the flavor SU(3)-breaking effect, we take the quark mass as $m_u = m_d = 0$ and $m_s \neq 0$ in our calculation.

We substitute the above propagator and the relevant photon DAs [17, 18] into correlation function (1) and integrate over the time-space coordinate x , we can obtain the expression of the amplitude $T((p-q)^2, q^2)$ up to twist-4 accuracy. The following taking place is a standard process in light-cone sum rules. The hadronic spectral function $\rho^h(s_1, s_2)$, which was used to control the contributions of excited states and of the continuum, can be approximately evaluated by using quark-hadron duality. To suppress the contributions of the excited and continuum states and of the subtraction terms in the hadronic representation, the double Borel transformation [24–26] about the variables $-q^2$ and $-(p-q)^2$ was introduced and the useful formulas for transformation as follow:

$$\mathcal{B}_{M_1^2} \mathcal{B}_{M_2^2} \left\{ \frac{\Gamma(\alpha)}{[-uq^2 - \bar{u}(p-q)^2]^\alpha} \right\} = \frac{(M^2)^{2-\alpha}}{M_1^2 M_2^2} \delta(u - u_0) \quad (11)$$

and

$$\mathcal{B}_{M_1^2} \mathcal{B}_{M_2^2} \left\{ \frac{1}{((p-q)^2 - m_V^2)(q^2 - m_P^2)} \right\} = \frac{1}{M_1^2 M_2^2} e^{-\frac{m_P^2}{M_1^2}} e^{-\frac{m_V^2}{M_2^2}}. \quad (12)$$

Here $M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}$, $u_0 = \frac{M_1^2}{M_1^2 + M_2^2}$, M_1^2 and M_2^2 are the Borel parameters associated with $-q^2$ and $-(p-q)^2$, respectively.

After the above lengthy calculation, we obtained the final result for the coupling $g_{VP\gamma}$:

$$g_{VP\gamma} = \frac{1}{\lambda_V \lambda_P} e^{\frac{m_V^2}{M_2^2} + \frac{m_P^2}{M_1^2}} \left(\frac{A_{VP} \langle \bar{q}q \rangle}{2} X + B_{VP} \langle \bar{s}s \rangle (X + B) \right), \quad (13)$$

where $X = A + I_G(u_0) + I_F(u_0)$,

$$A = -M^2 \chi \left[\varphi_\gamma(u_0) + \sum_k b_k e^{-\frac{s_0}{M^2}} \left(\frac{1}{2} \right)^k k! \sum_{n=0}^k \frac{(1-2u_0)^{k-n}}{(k-n)! n!} \sum_{j=0}^{k-n} \frac{(\frac{s_0}{M^2})^j}{j!} \right] + \frac{\mathcal{A}(u_0)}{4}, \quad (14)$$

$$\begin{aligned} B = & -\frac{m_s}{\pi^2} M^2 (1 - e^{-\frac{s_0}{M^2}}) (1 - \frac{\gamma_E}{2}) \\ & - \frac{f_{3\gamma} m_s \phi^{(a)}(u_0)}{2} - \frac{2 \langle \bar{s}s \rangle}{3} + \frac{2 \langle \bar{s}s \rangle}{3M^2} m_s^2 + \frac{\langle \bar{s}s \rangle}{3\pi^2 M^4} m_s^2 m_0^2, \end{aligned} \quad (15)$$

$$\begin{aligned} I_G(u_0) = & \int_0^{u_0} d\alpha_q \int_0^{1-u_0} d\alpha_{\bar{q}} \frac{1}{1 - \alpha_q - \alpha_{\bar{q}}} [\mathcal{T}_1(\alpha_i) - \mathcal{T}_2(\alpha_i) + \mathcal{T}_3(\alpha_i) - \mathcal{T}_4(\alpha_i) - \mathcal{S}(\alpha_i) \\ & - \tilde{\mathcal{S}}(\alpha_i)] - 2 \int_0^{u_0} d\alpha_q \int_0^{1-u_0} d\alpha_{\bar{q}} \frac{1 - u_0 - \alpha_{\bar{q}}}{(1 - \alpha_q - \alpha_{\bar{q}})^2} [\mathcal{T}_3(\alpha_i) - \mathcal{T}_4(\alpha_i) - \tilde{\mathcal{S}}(\alpha_i)] \end{aligned} \quad (16)$$

and

$$I_F(u_0) = - \int_0^{u_0} d\alpha_q \int_0^{1-u_0} d\alpha_{\bar{q}} \frac{1}{1-\alpha_q-\alpha_{\bar{q}}} [\mathcal{S}_\gamma(\alpha_i) + \mathcal{T}_4^\gamma(\alpha_i)] + 2 \int_0^{u_0} d\alpha_q \int_0^{1-u_0} d\alpha_{\bar{q}} \frac{1-u_0-\alpha_{\bar{q}}}{(1-\alpha_q-\alpha_{\bar{q}})^2} \mathcal{T}_4^\gamma(\alpha_i). \quad (17)$$

Here χ is the magnetic susceptibility of the quark condensate, which has been introduced in the pioneering work [27] for proton and neutron magnetic moments. In our next numerical analysis, we take $\chi(1\text{GeV}^2) = -3.15 \pm 0.3\text{GeV}^{-2}$ which was obtained by using QCD sum rules analysis of two-points correlation function [17]. $\varphi_\gamma(u)$ is the photon leading twist distribution amplitude, $\phi^{(a)}(u_0)$ is the photon twist-3 distribution amplitude, $\mathcal{A}(u)$, $\mathcal{T}_j(\alpha_i)$ ($j = 1, 2, 3, 4$), $\mathcal{T}_4^\gamma(\alpha_i)$, $\mathcal{S}(\alpha_i)$, $\tilde{\mathcal{S}}(\alpha_i)$, and $\mathcal{S}_\gamma(\alpha_i)$ ($i = q, \bar{q}, g$ and $\alpha_g = 1 - \alpha_q - \alpha_{\bar{q}}$) are the photon twist-4 DAs. Their detailed expression are given in Appendix A. $f_{3\gamma} = -(0.0039 \pm 0.0020)\text{GeV}^2$ [17] is the nonperturbative constant to describe the photon twist-3 DAs and γ_E is Euler constant. b_k are the coefficients of the leading twist-2 distribution amplitude $\varphi_\gamma(u)$ by exploited as a power series in $(1-u)$, $\varphi_\gamma(u) = \sum_k b_k(1-u)^k$. The coupling $g_{VP\gamma}$ for the $P \rightarrow V\gamma$ decay can also be calculated by the similar approach. A summary of the results is presented in Table 1.

$V \backslash P$	η		η'	
	A_{VP}	B_{VP}	A_{VP}	B_{VP}
ϕ	$m_\phi(e_u + e_d) \sin \beta \cos \varphi$	$-m_\phi e_s \cos \beta \sin \varphi$	$m_\phi(e_u + e_d) \sin \beta \sin \varphi$	$m_\phi e_s \cos \beta \cos \varphi$
ω	$m_\omega(e_u + e_d) \cos \beta \cos \varphi$	$m_\omega e_s \sin \beta \sin \varphi$	$m_{\eta'}(e_u + e_d) \cos \beta \sin \varphi$	$-m_{\eta'} e_s \sin \beta \cos \varphi$
ρ	$m_\rho(e_u - e_d) \cos \varphi$	0	$m_{\eta'}(e_u - e_d) \sin \varphi$	0

Table 1: The parameters for the coupling $g_{VP\gamma}$ in the radiative decays of the light mesons.

III. NUMERICAL ANALYSIS

Now, we present our numerical analysis of the coupling constants $g_{VP\gamma}$ for the $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ decays. In order to obtain numerical results of the sum rules from Eq.(13) and Table.1, we take the input parameters as usual: $m_\phi = 1.02\text{GeV}$, $m_\omega = 0.782\text{GeV}$, $m_\rho = 0.77\text{GeV}$, $m_\eta = 0.55\text{GeV}$, $m_{\eta'} = 0.958\text{GeV}$, $m_s = 0.156\text{GeV}$, $\langle \bar{q}q \rangle = -(0.24\text{GeV})^3$, $\langle \bar{s}s \rangle = 0.8\langle \bar{q}q \rangle$. The values of overlap amplitudes λ_V from Eq.(5) are $\lambda_\phi = 0.250 \pm 0.009\text{GeV}^2$, $\lambda_\omega = 0.162 \pm 0.004\text{GeV}^2$, $\lambda_\rho = 0.150 \pm 0.003\text{GeV}^2$ and their numerical analysis was presented in Appendix B. There haven't the numerical results for the overlap amplitudes λ_P from Eq.(6), but their expressions with the mixing angle φ were given by using QCD sum rules in Appendix B.

The following issue is to discuss how to reasonably choose the effective thresholds s_0, s_0^V, s_0^P for the sum rules of the coupling $g_{VP\gamma}$. s_0^V and s_0^P are the effective thresholds in the vector meson and the pseudoscalar meson channels, respectively. In general, their values ranges from the mass square of the ground state to the mass square of the first exciting state. According to data of PDG [28], we can find the first exciting states of the above meson. They are $\phi(1680)$ for ϕ meson, $\omega(1420)$ for ω meson, $\rho(1450)$ for ρ meson, $\eta(1295)$ for η meson, $\eta'(1405)$ for η' meson. While taking into account the especial property of the double spectral function at $s_1 = s_2$, we obtain the eventual range of the effective threshold in the double dispersion relation: $s_0 \in (1.04, 1.68)\text{GeV}^2$ for the coupling $g_{\phi\eta\gamma}$, $s_0 \in (1.04, 1.97)\text{GeV}^2$ for the coupling $g_{\phi\eta'\gamma}$, $s_0 \in (0.61, 1.68)\text{GeV}^2$ for the coupling $g_{\omega\eta\gamma}$, $s_0 \in (0.59, 1.68)\text{GeV}^2$ for the coupling $g_{\rho\eta\gamma}$, $s_0 \in (0.92, 1.97)\text{GeV}^2$ for the coupling $g_{\omega\eta'\gamma}$ and the coupling $g_{\rho\eta'\gamma}$. In the next analysis, the effective threshold s_0 will be strictly choose in the above ranges to fit the result of the sum rules and experimental data.

In Fig.1, we discussed the Borel window of our sum rules. Here the mixing angle φ is fixed at $\varphi = 40^\circ$, the Borel parameter M_2^2 of every coupling constant takes a defined value and the effective thresholds s_0 of the decay channel have different value which belong to the above discussing ranges. We find that there have a platform, where the coupling $g_{VP\gamma}$ is practically independent of the Borel parameter M_1^2 , $1.8\text{GeV}^2 \leq M_1^2 \leq 2.8\text{GeV}^2$ for the η channel and $1.5\text{GeV}^2 \leq M_1^2 \leq 2.5\text{GeV}^2$ for the η' channel. So our sum rules are reasonable and significative.

In Fig.2, we show theoretical and experimental values of the couplings $g_{VP\gamma}$ as functions of the $\eta - \eta'$ mixing angle φ with the defined s_0 , M_2^2 and the variable M_1^2 . The ranges of M_1^2 are the Borel windows from Fig.1. So theoretical predictions are presented in the shadows in Fig.2. By using the definition of $g_{VP\gamma}$ from Eq.(8), the decay widths of $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ are written as

$$\Gamma(V \rightarrow P\gamma) = \frac{\alpha g_{VP\gamma}^2 (m_V^2 - m_P^2)^3}{24 m_V^5}, \quad \Gamma(P \rightarrow V\gamma) = \frac{\alpha g_{VP\gamma}^2 (m_P^2 - m_V^2)^3}{8 m_P^5}, \quad (18)$$

where $\alpha = 1/137$ is the electromagnetic coupling constant. Comparing Eq.(18) with experimental data [28], we obtain the values of the coupling constants, which are presented in the dot-dashed curves in Fig.2. Finally, the pseudoscalar mixing angle φ from different $V - \eta, \eta'$ electromagnetic coupling processes and their average value are list in Table.2.

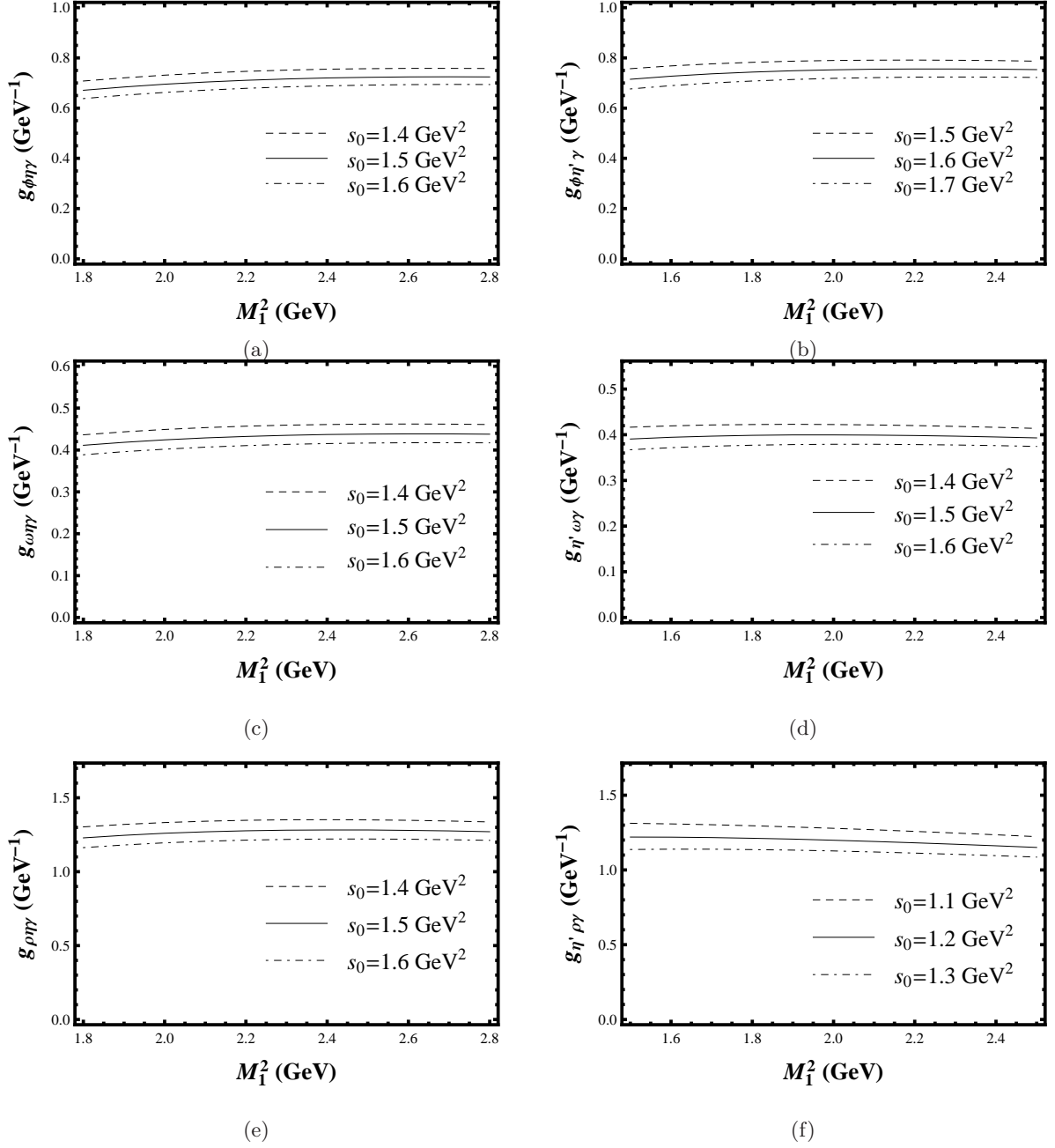


FIG. 1: The coupling constants $g_{VP\gamma}$ and $g_{PV\gamma}$ as a function of the Borel parameter M_1^2 for different values of the threshold parameters s_0 with defined M_2^2 . (a) $M_2^2 = 2.0 \text{ GeV}^2$; (b) $M_2^2 = 2.3 \text{ GeV}^2$; (c) $M_2^2 = 1.9 \text{ GeV}^2$; (d) $M_2^2 = 2.1 \text{ GeV}^2$; (e) $M_2^2 = 1.7 \text{ GeV}^2$; (f) $M_2^2 = 1.9 \text{ GeV}^2$.

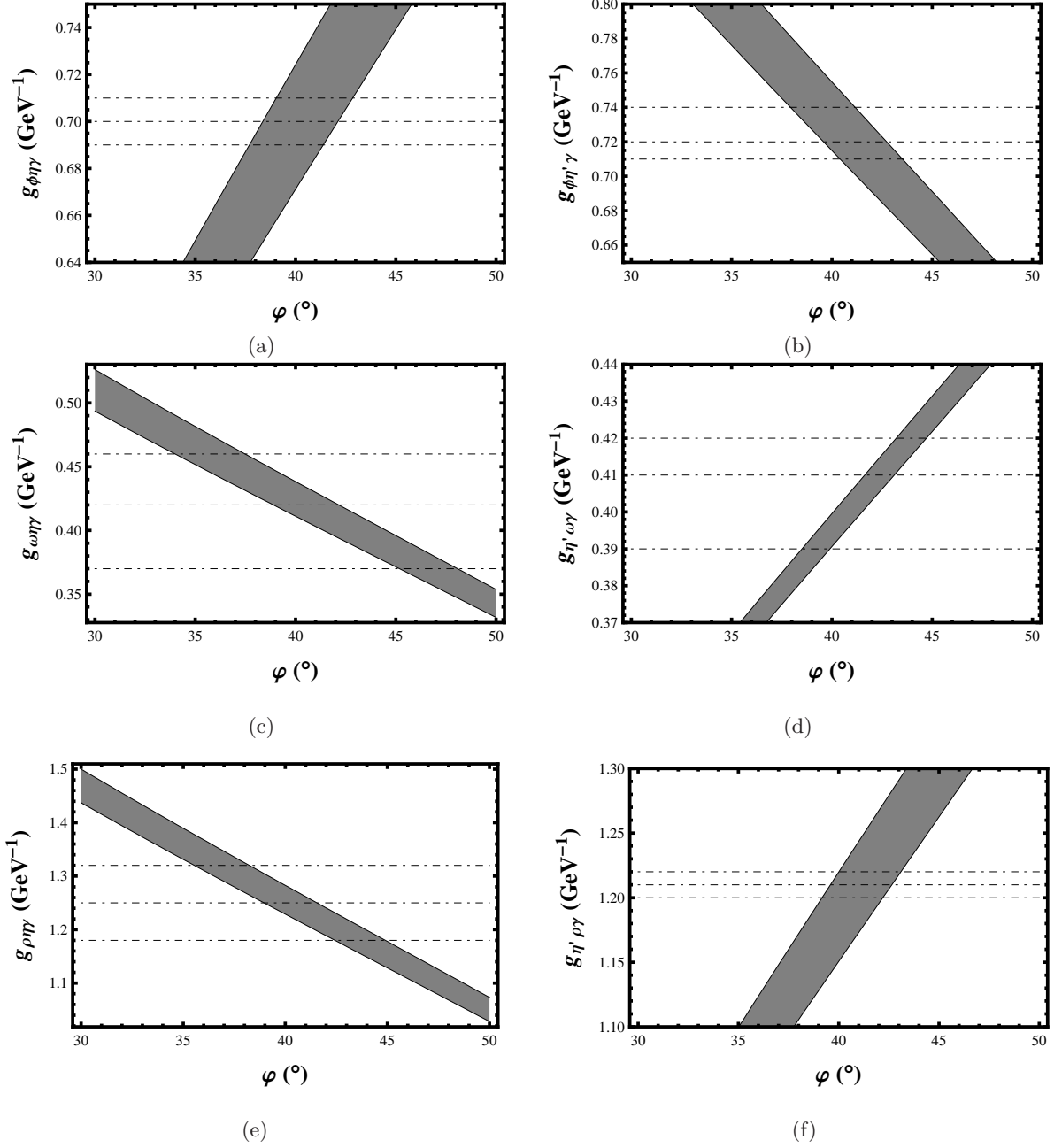


FIG. 2: Theoretical and experimental values of the couplings $g_{VP\gamma}$ as functions of the $\eta - \eta'$ mixing angle φ . The shadows are theoretical predictions according to Eq.(13) and the dot-dashed curves are the couplings extracted from experimental data [28] by Eq.(18). (a) $M_2^2 = 2.0\text{GeV}^2$ and $s_0 = 1.5\text{GeV}^2$; (b) $M_2^2 = 2.3\text{GeV}^2$ and $s_0 = 1.6\text{GeV}^2$; (c) $M_2^2 = 1.9\text{GeV}^2$ and $s_0 = 1.5\text{GeV}^2$; (d) $M_2^2 = 2.1\text{GeV}^2$ and $s_0 = 1.5\text{GeV}^2$; (e) $M_2^2 = 1.7\text{GeV}^2$ and $s_0 = 1.5\text{GeV}^2$; and (f) $M_2^2 = 1.9\text{GeV}^2$ and $s_0 = 1.2\text{GeV}^2$.

$V(P) \rightarrow P(V)\gamma$	$\phi \rightarrow \eta\gamma$	$\omega \rightarrow \eta\gamma$	$\rho \rightarrow \eta\gamma$	$\phi \rightarrow \eta'\gamma$	$\eta' \rightarrow \omega\gamma$	$\eta' \rightarrow \rho\gamma$	$\varphi_{av}(^\circ)$
$\varphi(^\circ)$	40.2 ± 0.7	41.2 ± 5.4	40.3 ± 3.4	40.8 ± 1.2	41.6 ± 2.4	41.1 ± 0.4	40.9 ± 0.5

Table 2: The mixing angle φ from different $V - \eta, \eta'$ electromagnetic coupling processes.

IV. CONCLUSIONS

We calculated the coupling $g_{VP\gamma}$ ($V = \phi, \omega, \rho$ and $P = \eta, \eta'$) of the $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ electromagnetic decays in the light-cone QCD sum rules. Comparing theoretical results and experimental data, we extracted a new pseudoscalar mixing angle $\varphi = 40.9 \pm 0.5^\circ$ in the quark-flavor basis. This result is in agreement with Ref.[6], where the average $\varphi = 39.3 \pm 1.0^\circ$. Recently, the KLOE Collaboration [29, 30] has measured the ratio $R_\phi = \mathcal{B}(\phi \rightarrow \eta'\gamma)/\mathcal{B}(\phi \rightarrow \eta\gamma) = 4.77 \times 10^{-3}$, the pseudoscalar mixing angle $\varphi = 41.4 \pm 1.6^\circ$ with the zero gluonium content for η' and $\varphi = 39.7 \pm 0.7^\circ$ with the gluonium content for η' . There is a little discrepancy between our theoretical results, $R_\phi = 4.85 \times 10^{-3}$ and $\varphi = 40.5 \pm 1.0^\circ$ from the $\phi \rightarrow \eta\gamma$ and $\phi \rightarrow \eta'\gamma$ decays, and the experimental results from KLOE. A possible reason is that we should consider the contribution from the gluonium content of η' meson in our calculation. This will be our next work.

Appendix A: The photon distribution amplitudes

In this section, the clear expressions for the photon distribution amplitudes are showed as[17, 18]

$$\begin{aligned}
\varphi_\gamma(u) &= 6u\bar{u}(1 + \varphi_2 \mathcal{C}_2^{\frac{3}{2}}(u - \bar{u})), \\
\phi^{(a)}(u) &= (1 - (2u - 1)^2)(5(2u - 1)^2 - 1)\frac{5}{2}(1 + \frac{9}{16}\omega_\gamma^V - \frac{3}{16}\omega_\gamma^A), \\
\mathcal{A}(u) &= 40u^2\bar{u}^2(3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2)[u\bar{u}(2 + 13u\bar{u}) \\
&\quad + 2u^3(10 - 15u + 6u^2)\ln(u) + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln(\bar{u})], \\
\mathcal{T}_1(\alpha_i) &= -120(3\zeta_2 + \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
\mathcal{T}_2(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)((\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)), \\
\mathcal{T}_3(\alpha_i) &= -120(3\zeta_2 - \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
\mathcal{T}_4(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)((\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)), \\
\mathcal{S}(\alpha_i) &= 30\alpha_g^2\{(\kappa + \kappa^+)(1 - \alpha_g) + (\zeta_1 + \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \\
&\quad + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\}, \\
\tilde{\mathcal{S}}(\alpha_i) &= -30\alpha_g^2\{(\kappa - \kappa^+)(1 - \alpha_g) + (\zeta_1 - \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \\
&\quad + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\}, \\
\mathcal{S}_\gamma(\alpha_i) &= 60\alpha_g^2(\alpha_q + \alpha_{\bar{q}})(4 - 7(\alpha_{\bar{q}} + \alpha_q)), \\
\mathcal{T}_4^\gamma(\alpha_i) &= 60\alpha_g^2(\alpha_q - \alpha_{\bar{q}})(4 - 7(\alpha_{\bar{q}} + \alpha_q)).
\end{aligned}$$

Here $\varphi_\gamma(u)$ is the photon leading twist distribution amplitude, $\phi^{(a)}(u_0)$ is the photon twist-3 DA, $\mathcal{A}(u)$, $\mathcal{T}_j(\alpha_i)$ ($j = 1, 2, 3, 4$), $\mathcal{T}_4^\gamma(\alpha_i)$, $\mathcal{S}(\alpha_i)$, $\tilde{\mathcal{S}}(\alpha_i)$, and $\mathcal{S}_\gamma(\alpha_i)$ ($i = q, \bar{q}, g$ and $\alpha_g = 1 - \alpha_q - \alpha_{\bar{q}}$) are the photon twist 4 DAs. The parameters appearing in the above DAs are given as $\varphi_2 = 0$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\omega_\gamma^V = 3.8$, $\omega_\gamma^A = -2.1$, $\zeta_2 = 0.3$, $\zeta_1^+ = 0$ and $\zeta_2^+ = 0$ at the scale $\mu = 1\text{GeV}$.

Appendix B: Calculation of Overlap Amplitude

In this section, we present the discussion of the overlap amplitudes for the vector mesons and pseudoscalar mesons. They are important input parameters for the coupling $g_{VP\gamma}$ in our sum rules. The overlap amplitudes λ_η , $\lambda_{\eta'}$ for pseudoscalar meson η and η' are given as[31]

$$\lambda_\eta = \lambda_\eta^q \cos \varphi - \lambda_\eta^s \sin \varphi, \quad \lambda_{\eta'} = \lambda_{\eta'}^q \cos \varphi + \lambda_{\eta'}^s \sin \varphi$$

in QCD sum rules, where

$$\begin{aligned}
(\lambda_P^q)^2 &= e^{\frac{m_P^2}{M^2}} M^4 \left\{ \frac{3}{8\pi^2 M^2} [1 - (1 + \frac{s_0^P}{M^2}) e^{-\frac{s_0^P}{M^2}}] + \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{8M^4} - m_q \langle \bar{q}q \rangle \right\} \frac{1}{M^4} + \frac{112\pi}{27M^6} < \sqrt{\alpha_s} \bar{q}q >^2 \}, \\
(\lambda_P^s)^2 &= e^{\frac{m_P^2}{M^2}} M^4 \left\{ \frac{3}{8\pi^2 M^2} [1 - (1 + \frac{s_0^P}{M^2}) e^{-\frac{s_0^P}{M^2}}] + \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{8M^4} - m_s \langle \bar{s}s \rangle \right\} \frac{1}{M^4} + \frac{112\pi}{27M^6} < \sqrt{\alpha_s} \bar{s}s >^2 \}.
\end{aligned}$$

Here s_0^P is the effective threshold and the pseudoscalar mixing angle φ will be determined by numerical analysis of $g_{VP\gamma}$.

With similar processes mentioned in the overlap amplitudes λ_P of pseudoscalar meson, the overlap amplitudes λ_V of vector mesons can be obtained and $\lambda_V^{q,s}$ are written as [26]:

$$\begin{aligned}
(\lambda_V^q)^2 &= m_V^2 M^2 e^{m_V^2/M^2} \left[\frac{1}{4\pi^2} (1 - e^{-s_0^V/M^2}) \left(1 + \frac{\alpha_s}{\pi} \right) + \frac{(m_u + m_d)}{2M^2} \langle \bar{u}u \rangle \right. \\
&\quad \left. + \frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a\mu\nu} \rangle}{12M^4} - \frac{112\pi \alpha_s \langle \bar{u}u \rangle^2}{81 M^6} + \frac{m_u^3 + m_d^3}{36M^8} \langle g_s \bar{u} \sigma_{\mu\nu} \frac{\lambda_a}{2} G^{a\mu\nu} u \rangle \right], \\
(\lambda_V^s)^2 &= m_V^2 M^2 e^{m_V^2/M^2} \left[\frac{1}{4\pi^2} (1 - e^{-s_0^V/M^2}) \left(1 + \frac{\alpha_s}{\pi} \right) + \frac{m_s}{M^2} \langle \bar{s}s \rangle \right. \\
&\quad \left. + \frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a\mu\nu} \rangle}{12M^4} - \frac{112\pi \alpha_s \langle \bar{s}s \rangle^2}{81 M^6} + \frac{m_s^3}{18M^8} \langle g_s \bar{s} \sigma_{\mu\nu} \frac{\lambda_a}{2} G^{a\mu\nu} s \rangle \right].
\end{aligned}$$

And then we take the input parameters: $m_\phi = 1.02 GeV$, $m_\omega = 0.782 GeV$, $m_\rho = 0.77 GeV$, $m_u = 0.005 GeV$, $m_d = 0.008 GeV$, $m_s = 0.156 GeV$, $\langle \bar{q}q \rangle = -(0.24 GeV)^3$, $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$, $\alpha_s = 0.5$, $m_0^2 = 0.8 GeV^2$, $\frac{\alpha_s}{\pi} G^2 = 0.012 GeV^4$, $\langle g_s \bar{s} \sigma_{\mu\nu} \frac{\lambda_a}{2} G^{a\mu\nu} s \rangle = m_0^2 \langle \bar{s}s \rangle$ into the above expressions and yield:

$$\lambda_\phi = \lambda_\phi^q \sin \beta + \lambda_\phi^s \cos \beta = 0.250 \pm 0.009 GeV^2,$$

$$\lambda_\omega = \lambda_\omega^q \cos \beta - \lambda_\omega^s \sin \beta = 0.162 \pm 0.004 GeV^2,$$

$$\lambda_\rho = \lambda_\rho^q = 0.150 \pm 0.003 GeV^2.$$

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